Augmented Taylor rule and independent prudential rule

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We address the question of the design of a Taylor-type rule that a central bank should be requested legally to follow as proposed by the economist John B. Taylor in 2014. We follow the recent development of *augmenting* the Taylor rule with a proxy for financial stability and focus on the credit spread. Then we compare our results with a *separated* financial stability reaction function and focus on a counter-cyclical prudential ratio. We incorporate the reaction functions in a New-Keynesian model including financial frictions à la Gertler and Karadi (2011) and perform our evaluation from the observed variations on the households welfare. With the hypothesis that it is not possible for the authorities to distinguish immediately between a non-financial and a financial shock, we recommend that the monetary policy reacts strongly to inflation and that the prudential authority reacts to the aggregate credit.

Key Words: augmented Taylor rule, counter-cyclical leverage ratio

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1. INTRODUCTION

In his testimony for the Committee on Financial Services before the United States House of Representatives in February 2014, John B. Taylor (the economist that devised the Taylor rule that can be used by central bankers) detailed his view on the legislation to impose on a central bank, remaining true to his idea in 1993 that operating a rule cannot be done by computer, but requires economic judgment from the authority:

> I have proposed that legislation be enacted requiring the Fed to adopt a policy rule–of its own choosing–for the instruments of policy, and that if and when the Fed deviates from its chosen rule, the Fed Chair would have to explain why in writing and in testimony [...].

In [Taylor 1993], John B. Taylor drew his work on the academic work that highlighted the benefit in terms of economic performance of rules versus discretion for the conduct of monetary policy facing inter-temporal inconsistency. This early work assessed monetary policies responding to exchange rate, money supply or price level and output gap. The latter prevailed.

There is still debate in the academic research about "optimal" simple reaction functions for a central bank, on the question of the dominance between Taylor or Wicksellian rules, where in the latter the short-term policy rate depends on deviations of the price level from a trend, [Giannoni 2014] concluding in favor of a Wicksellian rule augmented with a high degree of inertia in a forward-looking New-Keynesian model.

Aside these discussions, empirical exercises have estimated the coefficients that the Federal reserve historically attributed to inflation and output gap. A recent study by [Curdia et al. 2014] argued that a Wicksellian rule would actually better fit the data.

The latter methods can be challenged by indeterminacy issues that have been raised in [Cochrane 2011], arguing that regressions on empirical data don't allow to test the value of the coefficients of a Taylor-type reaction function of the Central Bank. This indeterminacy could raise the question of the "controllability" of the monetary policy. If the coefficients historically used by the monetary authority cannot be "monitored", then it would not make sense to impose a specific Taylor-rule on the central bank.

From the work of [Leeper 1991], the design of an *active* monetary rule would be coupled with a *passive* fiscal rule, or vice versa, and so part of the decision would be to decide which, from monetary to fiscal, to choose active and passive (or both). [Leeper 1991] derive a model where, for the price to be uniquely defined, one policy has to be active, whereas the other policy has to be passive to balance the inter-temporal government budget.

A further limitation is demonstrated in [Giannoni 2014], where if it acknowledges that Taylor rules are robust to model misspecifications. He finds that they are sensitive to the assumptions on the processes governing the shocks. In particular, the efficiency of a designed Taylor rule can degrade if the persistence of the shocks is higher than expected. We have to note, however that the study of [Giannoni 2014] is only limited to a simple model with a New Keynesian Phillips Curve and a Dynamic IS equation, but doesn't incorporate predetermined variables (as capital), nor other financial frictions than sticky prices. But as we will show in this paper, knowing the nature of the socks is fundamental and if the nature of the shocks cannot be properly assessed in real-time by the authority, it might impact the sign or the significance of a reaction recommendation as in the work of [Curdia and Woodford 2010]. In their work, if the nature of the shock is financial or non-financial, then the monetary authority should react positively or negatively to the credit spread.

Knowing the limitations, I want to assess John B. Taylor suggestion and I will still focus on a Taylor-type rule, with a de facto passive fiscal policy. I will suppose that the central bank can commit to a simple rule, and can be "controlled" by the agents that she is following her declared rule. As originally stressed in [Taylor 1993], the values and variables considered in the rule could depend on the economic circumstances. I assume that the central bank is benevolent, and follows the simple Taylor-type rule that maximizes welfare, if the optimal coefficient is finite. In our modeling, simplification of reality might imply that it would be optimal to have a coefficient tending to infinity. I will in my conclusion interpret this as "the central bank commits to react strongly to the variable".

In simple terms, in the following work, I address the following question: '**If we were to impose a simple Taylor-type rule on a central bank, should it be augmented with financial frictions or should we add an independent simple prudential rule?**'. I will aim at characterizing the **sign** of the policy response to that set of variables (inflation, output gap, credit spread, credit aggregate) and the **magnitude** that response should have.

The question of the optimal Taylor-type rule coefficient has been extensively explored in the work of Stephanie Schmitt-Grohe and Martin Uribe. Drawing from their early work on second-order approximation to policy function in DSGE [Schmitt-Grohe and Uribe 2004], they explored optimal simple and implementable monetary and fiscal rules [Schmitt-Grohe and Uribe 2007]. Comparing various monetary policy set-ups to the Ramsey-optimal policy, they found that an optimal simple rule should respond to inflation, with only a minor importance on the value of the coefficient as long as it verifies the Taylor principle. They also conclude that it is detrimental for the policy rule to respond to output fluctuations and that the fiscal policy should be *passive*.

I base my research on the work of [Schmitt-Grohe and Uribe 2007], starting from Section 3, I study optimal policy in a world where there are no subsidies to undo the distortions created by imperfect competition, where there is capital accumulation, where nominal rigidities induce inefficiencies even in the long run. But I depart from their model as on one hand, I do not cover the demand for money, nor the potential distortions from the fiscal policies. On the other hand, I follow the model of [Gertler and Karadi 2011] as it allows me to consider the impact of **leverage ratio** constraints on financial intermediaries. I will augment the simple Taylor rule with financial frictions and compare these gains to an independent prudential policy.

The benefits of augmenting the Taylor rule with a measure of financial frictions have been assessed in [Faia and Monacelli 2007] where they found that the interest rate could counter increases in asset prices and credit frictions, but the gains were marginal to dampening distortions from price stickiness. Hence they recommend the use of one instrument for one policy goal. In [Curdia and Woodford 2010], the two measures of financial frictions explored were the credit spread and a measure of aggregate credit. With their model, they found that it might be optimal to introduce the credit spread in the Taylor rule, but as response to financial disturbances, this is more nuanced when it comes to non-financial disturbances.

This motivates our research, to further explore whether to *augment* the Taylor rule with financial frictions or to have a *separate*

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simple prudential rule reacting counter-cyclically to financial frictions.

As reviewed in [Carre et al. 2015], I can propose several factors and proxies that can be used to model financial stability, from which we can chose to design an independent prudential policy or augment the Taylor rule:

Factor	**Proxy**
credit	credit spread
banking sector	loan volume, bank bankruptcy
asset prices distortions	bubbles formation
exchange rates	volatility in the foreign exchange markets
interbanking sector	OIS
systemic risk	systemic risk measure

Starting from the simplest for of a Taylor rule (Equation 16, Appendix A), we can:

(1) *augment* the rule with a combination of the selected proxies from above (Equation 17, Appendix A is an example with the credit spread proxy)

(2) *separate* the monetary rule from a prudential rule that can affect:
   (a) lenders, e.g. with a counter-cyclical prudential ratio constraint
   (b) borrowers, e.g. with Loan to Income (LTI) or Loan to Value (LTV) constraints
   (c) capital fluxes, e.g. with taxes

The objective can be measured from the effects on welfare :

—with or without financial stability consideration in the welfare computation

—computed only for consumers or for different types of agents (consumers, entrepreneurs, bankers...)

In this work, I focus on the bank leverage as a macroprudential instrument, where there will be no distinction between banks (nor their exposure to risk). In [Berrospide and Edge 2010], they noted that since observed, the bank leverages for the United-States ranged between 8 and 12.5. Which means that there is approximately a 50% historically observed range of variations for the leverage.

In early work of [Peek and Rosengren 1991], they used a simple model of bank's balance sheets with the bank capital (K), bank reserves at the central bank (R) and loans (L) to define a capital-asset ratio μ as the inverse of the bank leverage ϕ as in Equation 1. They find a significant negative effect of the leverage ratio on the loan growth for small and large commercial banks. They work illustrated that when financial intermediaries equity increases, the aggregate level of credit in the economy increases as well.

$$\phi = \frac{1}{\mu} = \frac{R+L}{K} \tag{1}$$

I will start my investigation were the leverage is binding and set directly by the prudential authority as a modification to the [Gertler and Karadi 2011] model. This presents the drawback, however, that I abandon the micro-foundation of the model that introduced endogenously a constraint on banks holding capital, with the banker choosing to divert a portion of the fund or not.

In [Angelini et al. 2014b], they add a cost that banks incur when varying the capital-assets ratio. A cost that then impacts the interest rate when lending to households and entrepreneurs.

In [Kannan et al. 2012], they introduce the macro-prudential instrument directly as a coefficient of the private lending rate. The prudential authority is then acting on the lending condition channel.

The last two methods will allow us to keep the micro-foundation for endogenous bank capital constraint from the [Gertler and Karadi 2011] paper, while adding a time-varying prudential measure on top. As in [Hubbard et al. 2002], the interest rate granted by a financial intermediary depends on the balance sheet constraint, at that time, on the financial intermediary. The more constrained, the higher the interest rate on the granted loan as the financial intermediary becomes more and more reluctant to add risk to its portfolio.

Finally, a missing aspect of the model is the impact of the evolution of credit in the economy on the systemic risk at the micro level. We do not assess the risk correlation among banks, nor the negative impact on the agents' welfare of the systemic risk build up.

In this paper, I will restrict my investigation to the **credit spread proxy** that augment a simple Taylor rule, and to the **counter-cyclical prudential ratio** to design a *separate* prudential reaction function; simulate the economy with Dynamic Stochastic General Equilibrium models; a simple one à la Jordi Gali (2008), and also additional financial frictions à la Gertler and Karadi (2011); and compute the welfare (second order) considering only the consumers.

2. A MODEL À LA GALI 2008

First we follow a basic New Keynesian model à la [Gali 2008] Chapter 3 . This is to our knowledge one of the simplest New Keynesian models, all the variables are presented in Appendix D where we also derive the model in first then second order, with namely the New Keynesian Phillips Curve and the Dynamic IS Curve. We use the same calibration as in the book. The economy is modeled with product differentiation, monopolistic competition and sticky prices (probability α), with a model of staggered price setting as in [Calvo 1983]. Money, in this model, provides no utility to the agents and is only a unit in which goods, labor and assets are quoted.

The representative household is to maximize its utility that we model as follow:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

s.t.

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

Where

—β the time discount rate and $\rho = -log\beta$
—C_t is a consumption aggregator of the CES (constant elasticity of substitution σ) form as in [Dixit and Stiglitz 1977]
—P_t the aggregate price index and the inflation $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$
—N_t are the hours worked (with φ the Frisch elasticity of labor supply)
—B_t are one period bonds
—Q_t is the bond price and $i_t = -logQ_t$
—W_t the nominal wage
—T_t a lump-sum component of income (tax and/or dividend from firms)

The monetary authority is setting its key interest rate i_t as a reaction to inflation ϕ_t and the log-deviation of output form the flexible price output level $\tilde{y}_t$ and a innovation ν_t:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t$$

2.1 Calibration

The model is calibrated as in the book [Gali 2008].

$\sigma = 1$	log utility
$\phi = 1$	unitary Frisch elasticity
$\phi_\pi = 1.5$	inflation feedback Taylor Rule
$\phi_y = .5/4$	output feedback Taylor Rule
$\theta = 2/3$	Calvo parameter
$\rho_\nu = 0.5$	autocorrelation monetary policy shock
$\rho_A = 0.9$	autocorrelation technology shock
$\beta = 0.99$	discount factor
$\eta = 4$	semi-elasticity of money demand
$\alpha = 1/3$	capital share
$\epsilon = 6$	demand elasticity

2.2 First order technology shock simulation

When log-linearized, we simulate a technology shock to the model as in Figure 1, setting the coefficients $\phi_y = 0.4/5$ and $\phi_\pi = 1.5$.

A negative technology shock lead the central bank to raise the interest rate as a reaction to an increase in inflation. In this model, the monetary policy is also reacting to the output gap (which would be optimal if it was measurable).

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The welfare loss is coming from two factors which are necessary to study monetary policy from a normative perspective:

—monopolistic competition between firms
—sticky prices

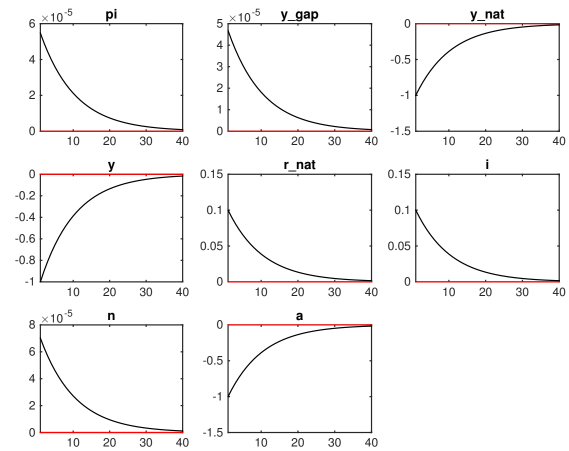

Fig. 1. Model á la Gali 2008, positive technology shock

2.3 Direct welfare loss computation

As demonstrated in [Gali 2008], we can approximate the welfare loss function in case of technology shocks:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda} \pi_t^2 + (\sigma + \frac{\varphi + \alpha}{1-\alpha}) \tilde{y_t}^2] \tag{2}$$

From Appendix D Equations 18, 20 and 19 we obtain the relationships between the variances:

$$Var(\tilde{y}_t) = [\sigma \Psi_{ya}^n (1-\rho_a)(1-\beta\rho_a)]^2 \frac{1}{1-\rho_a^2} Var(\epsilon_t^a)$$

$$Var(\pi_t) = [\sigma \Psi_{ya}^n (1-\rho_a)\kappa]^2 \frac{1}{1-\rho_a^2} Var(\epsilon_t^a)$$

We can compute the **Loss Function** so that the variance of the output gap and the inflation are the "only" variables to take into consideration:

$$\mathbb{L} = \frac{1}{2} [\frac{\epsilon}{\lambda} Var(\pi_t) + (\sigma + \frac{\varphi + \alpha}{1-\alpha}) Var(\tilde{y}_t)] \tag{3}$$

With this method, no extensive second order simulation has to be perform to obtain a result.

Note however that if we assume that the central banker would be able to directly monitor $\tilde{y}_t$, the optimal coefficients of the Taylor rule differ. But the computation of the latter would mean that the central banker is able to observe the stochastic joint distribution of the shocks in order to abstract from their effect.

We calculate the welfare loss and update the table 4.1 from [Gali 2008] with different macroeconomic variables monitored by the central bank:

—$\hat{y}_t \equiv log(Y_t/Y)$ which is the log-deviation of output from its steady state

—$\tilde{y}_t \equiv y_t - y_t^{\text{nat}}$ which is the log-deviation of output form the flexible price output level (natural level of output)

—$L_{\hat{y}}$: Loss with $\hat{y}_t$ monitored as in [Gali 2008]

—$L_{\tilde{y}}$: Loss with $\tilde{y}_t$ monitored

The monetary policy rule is modeled:

$$i_t = -log(\beta) + \phi_\pi(\pi_t - 1) + \phi_y log(Y_t/Y) \tag{4}$$

We obtain the following welfare losses:

ϕ_π	1,5	1,5	1,5	5	5
ϕ_y	0	0,125	1	0	2
$L_{\hat{y}}$	0.080291	0.304228	1.923953	0.002154	0.47874
$L_{\tilde{y}}$	0.080291	0.060094	0.015900	0.002154	0.001086

When the central banker monitors $\hat{y}_t$, we find exactly as in [Gali 2008] that it is optimal never to react to the observed output gap. This conclusion is, however, not valid if it became possible to base the monetary policy on $\tilde{y}_t$, where we find a minimum welfare loss for $\phi_\pi \to \infty$ (and/or) $\phi_y \to \infty$. This raises questions on the robustness of the simple rule used in this model.

We follow the argumentation in [Woodford 2001] that

> this is not an appealing policy proposal, as even small deviations from the idealized assumptions listed could be quite problematic. In practice, small errors in the inflation and output-gap measures available to the Fed in real time would surely lead to violent interest-rate volatility under such a rule.

Hence with these first results, we can say that it is optimal for the central bank not to react to the output gap and to react *reasonably* to inflation variations. At this stage of my investigation, I am not able to precisely define a *reasonable* reaction to inflation.

2.4 Second order approximation

When computing welfare, at least a second order approximation should be used.

In order to be able to simulate our model and evaluate the welfare loss, we have to resort to a second order derivation as presented in [Schmitt-Grohe and Uribe 2004], hence we have to derive a model avoiding first order log-linearization. The model is derived in Appendix D.1.

At this stage, with this model, it is not yet possible to augment the Taylor rule as no credit frictions are modeled.

I use the policy matrices from Dynare and compute the second order derivation of the model (with $\otimes$ the Kronecker product):

$$y_t = y^s + 0.5\text{ghs2} + \text{ghx}y_{t-1}^h + \text{ghu}u_t + 0.5\text{ghxx}(y_{t-1}^h \otimes y_{t-1}^h) \\ +0.5\text{ghuu}(u_t \otimes u_t) + \text{ghxu}(y_{t-1}^h \otimes u_t)$$

with a distinction to be made between the decision rules order and the declaration order in the above formula (cf. Dynare manual).

We simulate and "scan" ϕ_π from 0 to 20 and ϕ_y from $0.5/4$ to 10. I find the optimal value for $\phi_\pi \to \infty$ and $\phi_y \to 0.5/4$ as illustrated in Figure 2.

This confirms the results found with the quadratic loss calculations and validates our method mentioned above, deriving the model in second order and using the policy matrices extracted from Dynare and then computing the welfare by iterations. Furthermore, our results are in line with the literature we mentioned in the introduction, namely the work of [Schmitt-Grohe and Uribe 2007]. Where it is detrimental for the central bank to react to the output gap, but positive to react to inflation.

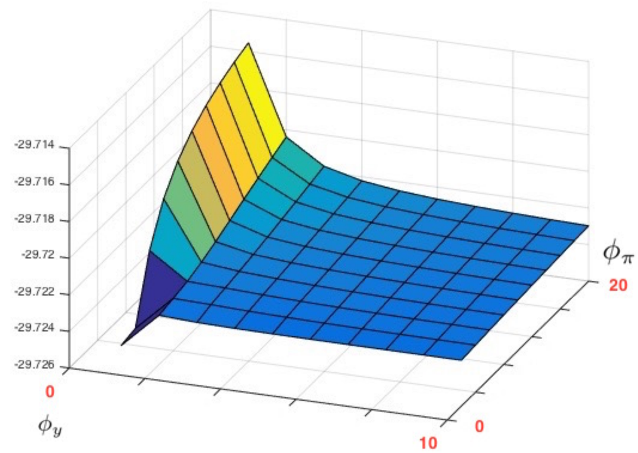

Fig. 2. Model á la Gali 2008, positive technology shock, welfare impact of the inflation and output coefficient of the Taylor rule

This was the first step: welfare loss computation for the economy with an "oversimplified" model. The next step is to introduce financial intermediaries frictions (we restrict our study to credit spread) and compare an *augmented* Taylor rule taking into account the credit spread with a *independent* macro-prudential rule (we restrict our study to counter-cyclical prudential ratios).

3. A MODEL OF UNCONVENTIONAL MONETARY POLICY À LA GERTLER AND KARADI (2011)

3.1 Modelling frictions in capital markets

To introduce financial intermediaries frictions, I decided to work on the model of [Gertler and Karadi 2011] for two main reasons:

—It is a model isomorphic to [Christiano et al. 2005] and [Smets and Wouters 2007], adding **frictions in capital markets** through financial intermediaries facing **endogenous** balance sheet constraints. This balance sheet constraints are in my view key to capture the difficulties in the monetary transmission to the real economy after 2007-09. The reason for banks to hold capital is not coming from prudential rules (which could be circumvented by financial engineering), but rather from the market as detailed in Appendix E.5 Equation 21.

—It is related to the financial accelerator model in [Bernanke et al. 1999] in which frictions were on non-financial firms. In [Gertler and Karadi 2011], frictions are now on financial intermediaries. This should be more adequate to model phenomenon of depression in financial activities as observed after 2007-09.

In the original model, there are six frictions on the six types of agents illustrated in Figure 3 with the following frictions:

—θ probability that a banker in a given period stays banker in the next period.

—ϕ_t leverage ratio

—U_t utilisation rate of capital (it is costly to vary the utilisation rate)

—ξ_t quality of capital, source of exogenous variation in the value of capital

—I_t gross capital created (with flow adjustment costs associated with producing new capital)

—τ central bank credit involves an efficiency cost of τ per unit supplied. Expenditure on government intermediation (cost to identify preferred private sector investments, etc.).

—$1-\gamma$ is the probability with which a retail firm can readjust its prices (as per [Calvo 1983])

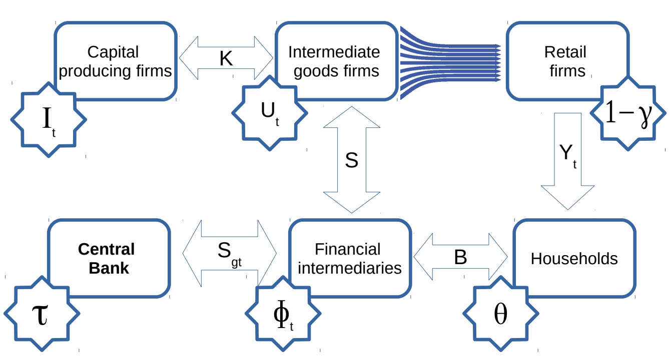

Fig. 3. Frictions in the [Gertler and Karadi 2011] model.

3.2 A simplified model

In the following, we simplify the Gertler and Karadi (2011) model:

(1) Households
—we drop the habit formation consideration

(2) Financial intermediaries
—no change

(3) Credit policy
—not present in our simplified model

(4) Intermediate goods firms
—we change the cost of utilization rate of capital to a constant δ that can be interpreted as the depreciation rate of capital

(5) Capital producing firms
—no change

(6) Retail firms
—we ignore the measure of price indexation γ_p set to null

(7) Government policy
—we ignore the government budget constraint and consider a government that chooses a monetary policy to minimize the credit spread

This simplified model is solved in Appendix E.

3.3 Presentation of the model

The [Gertler and Karadi 2011] model is a quantitative monetary DSGE model. The **households** provide work L_t for which they get the real wage W_t, they get a retribution R_t for overnight lending B_t. They own both non-financial and financial firms and receive net payouts Π_t and pay lump sum taxes T_t. They consume C_t. The households maximize the welfare

$$\log(C_t) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}$$

with a discount factor β and the budget constraint

$$C_t = W_t L_t + \Pi_t + R_t B_t - B_{t+1}$$

The **financial intermediaries** are composed of bankers, which are a fraction $1-f$ of household members and stay bankers the following period with a probability θ. Each new banker get start-up funds from his household and each exiting banker bring their retained earnings to their households. At the start of a period t, a banker j has equity N_{jt} and obtains B_{jt+1} as deposits from households and emits S_{jt} financial claims on non-financial firms with a relative price of Q_t, on which he earn the stochastic return R_{kt+1}. An intermediary balance sheet is $Q_t S_{jt} = N_{jt} + B_{jt+1}$ and its equity evolves as: $N_{jt+1} = (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} B_{jt+1}$. The banker's objective is to maximize it present expected discounted terminal wealth V_{jt}, given the stochastic discount factor $\beta^{i+1}\Lambda_{t,t+1+i}$:

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} V_{jt+i}$$

In order to create an endogenous capital constraint on the financial intermediaries, at each period, the banker can choose to divert a fraction λ of the funds back to his household (lost for the depositors). There is thus an incentive constraint: $V_{jt} \geq \lambda Q_t S_{jt}$. When the incentive binds, we can re-express it as $Q_t S_{jt} = \frac{\eta_t}{\lambda - \nu_t} N_{jt} = \phi_t N_{jt}$. Where we can interpret ν_t as the expected discounted marginal gain to the banker for expanding his assets, η_t the expected discounted value of having another unit of equity N_{jt}, and

ϕ_t the private leverage ratio. The model is calibrated so that the constraint always binds within a local region of the steady state. We sum across individuals to obtain: $Q_tS_t = \phi_tN_t$. In the model, variation in aggregate equity N_t will induce fluctuations in overall asset demand by intermediaries. A fraction θ of bankers survive over a period and a new banker bring back a fraction ω of the funds that were brought back the previous period. Hence, the equity evolves with the motion and ω pins down the steady-state value of the leverage ratio:

$$N_t = \theta[(R_{kt} - R_t)\phi_{t-1} + R_t]N_{t-1} + \omega Q_t S_{t-1} \quad (5)$$

Competitive **intermediate goods firms** acquire capital K_{t+1} for use in production, at price Q_t. To finance its capital acquisition the firm issues S_t claims equal to the number of units of capital acquired and prices each claim at the price Q_t. Then by arbitrage $Q_tK_{t+1} = Q_tS_t$. The production function is $Y_t = A_t(\xi_tK_t)^\alpha L_t^{1-\alpha}$ where the shock ξ_t is meant to provide a simple source of exogenous variation in the value of capital.The firms maximizes its profits and pins down the return R_{kt+1}.

The capital depreciate by δ. At the end of a period, it is bought by **capital producing firms** form the intermediate goods firms at the end of each period and it is refurbished before being resold to intermediate goods firms, to maximize their present discounted profits $(Q_t-1)I_{nt} - f(\frac{I_{nt}+I}{I_{nt-1})+I})(I_{nt}+I)$. The gross capital created during this operation is I_t (I its steady state value) and the net capital created is $I_{nt} \equiv I_t - \delta\xi_tK_t$. For the adjustment costs, following [Christiano et al. 2014], we define:

$$\begin{aligned} f(x_t) &\equiv S(x_t) \\ &= \frac{1}{2}\left\{\exp\left[\sqrt{S''}(x_t - x)\right] + \exp\left[-\sqrt{S''}(x_t - x)\right] - 2\right\} \end{aligned}$$

where $x_t \equiv I_t/I_{t-1}$ and x denotes the steady state value of x_t.

Retail firms package intermediate goods into a CES composite

$$Y_t = \left[\int_0^1 Y_{ft}^{(\varepsilon-1)/\varepsilon} df\right]^{\varepsilon/(\varepsilon-1)}$$

with ϵ the elasticity of substitution between goods and where Y_{ft} is a retailer f output. A Retailer maximizes its discounted profit knowing it has, at each period, the probability γ not to be able to freely set its price, and facing a price P_{mt} from the intermediate goods firms:

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[P_t^* - P_{mt+i}\right] Y_{ft+i}$$

The output is shared between consumption, investment (and its adjustment cost). That is the economy-wide **resource constraint**:

$$Y_t = C_t + I_t + f\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right)(I_{nt} + I_{ss})$$

Finally, the **monetary policy** is a Taylor rule and the **Fisher relation** links the nominal and real interest rate as describe in Appendix A.

3.3.1 *How we propose to further study* ***unconventional*** *policies.* In the original paper, to model unconventional monetary policy, they studied the injection of credit in the private sector by the central bank in response to the credit spreads according to a rule:

$$\psi_t = \psi + E_t[(\log R_{kt+1} - \log R_{t+1}) - (\log R_k - \log R)] \quad (6)$$

In the following, we set a path to challenge the welfare improvement introduce by the rule of Equation 6. We consider this *unconventional* monetary policy as an urgency measure reacting to an unexpected 2007-09 crisis. This is referred to as *mopping up* or *cleaning up afterwards* a crisis has burst. We intend to explore *leaning against the wind* strategies, that rather prevent crises from occurring or moderate their magnitudes. Furthermore, when the central bank directly lend to private companies, if incurs the cost τ. It is hard to rely on a proper calibration for τ as this doesn't capture the loss of welfare that can stem from *blunt* projects selection. For instance, in Europe, more than 60% of the employment in the non-financial business economy is provided by SMEs, the latter are also more dynamic in creating employment. But it is difficult for a central bank to navigate through the information of SMEs and this is rather the expertise of local (private) bankers. In the following, I study long term solutions and rules, and especially a contra-cyclical prudential ratio introduced in Section 4.

3.4 Calibration

We calibrate the model as in the original [Gertler and Karadi 2011]. This calibration could be later challenge when we temporary get rid of the micro-foundation and introduce a blunt prudential constraint in Section 4.1, but we keep the same parameters all along for simplicity:

α	0.330	capital share in production
β	0.990	discount factor
χ	3.409	disutility of labor
δ	0.025	capital depreciation rate
γ	0.779	probability of retailer not changing prices
κ_y	0.5/4	output gap coefficient of the Taylor rule
λ	0.381	fraction of funds divertable by the banker
μ	1.315756	retail firm markup
ω	0.002	household transfer fraction to entering bankers
φ	0.276	inverse Frisch elasticity of labor supply
ρ	0.8	smoothing parameter of the Taylor rule
ρ_a	0.9	autocorrelation param. for productivity shock
ρ_ξ	0.90	autocorrelation param. for capital quality shock
S'	5	first derivative of capital adjustment costs
σ_a	0.01	productivity shock volatility
θ	0.972	survival rate of banker

3.5 Predetermined variables

In this model, out of 24 variables, two are predetermined (chosen in t):

—B_{t+1} the level of debt

—K_{t+1} the level of capital

K_{t+1} remains in the equations used for the model.

This is a fundamental difference with the model of [Gali 2008], as derived in Appendix D, no predefined variables are left. In this [Gertler and Karadi 2011] model, there is endogenous investment. As shown in [Dupor 2001], this can change the uniqueness/indeterminacy results from the value of κ_π in Section 3.6 Equation 8.

3.6 Negative productivity shock

We assume the following AR(1) for the productivity evolution:

$$A_t = A_{t-1}^{\rho_A} exp(-\varepsilon_t^A) \quad (7)$$

If we take the same monetary policy rule as in the paper:

$$i_t = (1-\rho)[i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho i_{t-1} + \varepsilon_t^m \quad (8)$$

We obtain the following results Figure 4 for a negative technology shock (on A_t). When a negative productivity shock occurs, the marginal costs for the firms increase, hence the latter have to raise their price and the inflation goes up. To fight that inflation, the monetary authority raises its key rate (the interest rate). In order to compensate for the negative shock, workers work more hours, which lower their marginal product with respect to the marginal product of capital, hence the **credit spread increases**.

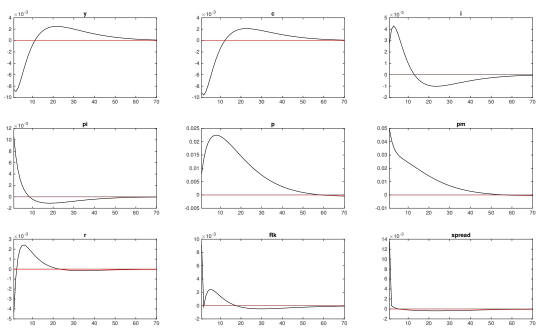

Fig. 4. Simplified Gertler and Karadi 2011, conventional monetary policy

3.7 Finance premium cyclicality

In [Faia and Monacelli 2007], they built an economy with nominal rigidities and credit-market imperfections and a monetary policy that follows a Taylor-type interest rate rule. They found a behaviour of the finance premium ("cost of external finance in excess of the safe rate of return") that is *countercyclical* which fit the empirical evidence as in [Levin et al. 2004]. In our simulations, we obtain also a countercyclical behavior of the finance premium, in other words, the credit spread increases when total factor productivity undergo a negative shock.

3.8 Welfare evaluation and optimal monetary policy

A complete and progressive description of the Taylor rules we use in this section and the following, with an emphasis on the financial stability variable ω_t is given in Appendix A.

As in [Faia and Monacelli 2007], our first goal is to find the optimal monetary policy coefficients ϕ_π and ϕ_y with or without the credit spread term and its coefficient ϕ_ω in the following monetary policy equation:

$$i_t = [i + \phi_\pi \pi_t + \phi_y(\log Y_t - \log Y_t^*) - \phi_\omega(\log \omega_t - \log \omega_t^*)] \quad (9)$$

We cannot estimate the welfare directly in Dynare as we should not rely on the first order approximation used in the "model" part of the file.

Once derived with second order approximation, it is not possible to use the Dynare "osr" function, following [Woodford 2003]. As to the best of our knowledge, the OSR function only works with first order approximation (cf. Michel Juillard answer in http://www.dynare.org/phpBB3/viewtopic.php?f=1&t=5308).

3.9 Second order derivation of the model

We derive the model with a second-order approximation and the differences with the first order are listed in Appendix H.

When simulating this model, we obtain very similar results to those found in section 3.6 as shown in Figure 5. When a negative productivity shock occurs, the marginal costs for the firms increase, hence the latter have to raise their price and the inflation goes up. To fight that inflation, the monetary authority raises its key rate (the interest rate). In order to compensate for the negative shock, workers work more hours, which lower their marginal product with respect to the marginal product of capital, hence the **credit spread increases**.

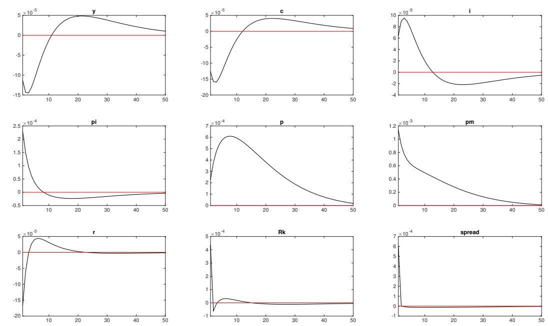

Fig. 5. Simplified Gertler and Karadi 2011, conventional monetary policy, second-order derivation

3.10 Optimal Taylor rule coefficients

When deriving this model, we have three periods: $t-1$, t and $t+1$ hence we cannot use the [Schmitt-Grohe and Uribe 2004] method. Instead, we use the policy matrices from Dynare and compute the second order approximation of the model (with $\otimes$ the Kronecker product):

$$y_t = y^s + 0.5\text{ghs2} + \text{ghx}y_{t-1}^h + \text{ghu}u_t + 0.5\text{ghxx}(y_{t-1}^h \otimes y_{t-1}^h) + 0.5\text{ghuu}(u_t \otimes u_t) + \text{ghxu}(y_{t-1}^h \otimes u_t)$$

with a distinction to be made between the decision rules order and the declaration order in the above formula (cf. Dynare manual).

3.10.1 *Non-financial shocks.* We simulate and "scan" for the optimal values of ϕ_π, ϕ_y and ϕ_ω as in Equation 9 of Section 3.8, with the credit spread: $\omega_t = R_{Kt} - R_t$..

When we "scan" simultaneously for all three coefficients, we obtain: $\phi_\pi \to \infty$ and $\phi_y \to 0$ and $\phi_\omega \approx -0.02$. When setting the value of $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ and scanning the credit spread coefficient in the negative value range (before the model's impulse response functions explode), we obtain an optimum for $\phi_\omega = -0.02$ as illustrated in Figure 6.

We compare it to what is found in [Carre et al. 2015] displayed in Figure 7. The authors compare a meta-analysis from academic research on augmented Taylor-rule coefficients. In the literature, however, the method to choose the coefficients are sometimes based on empirical data, or estimated historical central bank reaction function (which then depends on the region considered), and never take into account the welfare loss from financial instability (we do not either). This relates to the systemic risk impact on the agents'

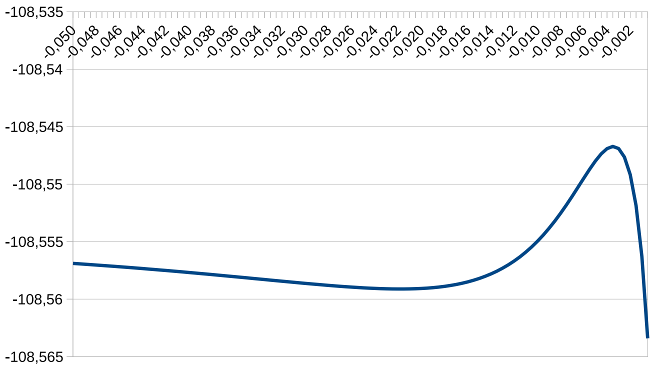

Fig. 6. Simplified Gertler and Karadi 2011, augmented Taylor rule, welfare evolution wrt credit spread coefficient - without prudential influence on the private loan rates

welfare we mention in Section 5. When comparing to the literature, our optimal reaction to the credit spread is of the lower range.

Tableau 5. *Principales statistiques descriptives*

Variable	Moyenne	Médiane	Écart type	Minimum	Maximum
α_s	0,40	0,20	0,53	0,00	2,50
α_π	2,98	2,00	3,16	0,50	18,00
α_y	1,51	0,25	4,03	0,00	21,60

Fig. 7. Optimal Taylor rule coefficients overview from [Carre et al. 2015]

In our case, the optimal value of the financial stability coefficient of the Taylor rule is small and negative. This is a similar result as in [Curdia and Woodford 2010] where the optimal Taylor rule response to *nonfinancial* disturbances (the productivity factor A_t in our case) is slightly negative, depending on the persistence of the shock ρ_A in Equation 7 in Section 3.6. We illustrate the impact of the persistence of the shock on the optimal value of ϕ_ω in Figure 8. As expected, the lower the persistence of the shock, the limited welfare loss. As opposed to [Curdia and Woodford 2010], the optimal value of our optimal ϕ_ω remains negative and doesn't venture into positive territory.

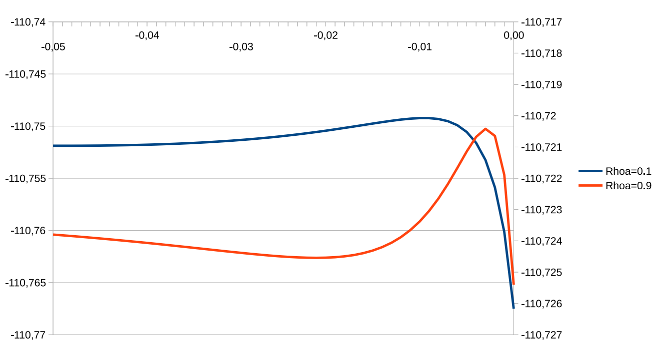

Fig. 8. Simplified Gertler and Karadi 2011, augmented Taylor rule, impact of the persistence of the technology shock on the optimal value of the credit spread coefficient ρ_A (0.1 right axis, 0.9 left axis)

We observe the interest rate reaction with two different persistence of the technology shock $\rho_A = 0.1$ and 0.9. We compare the optimal augmented Taylor rule reaction to a non-augmented Taylor rule reaction. In both cases (of persistence value), the Taylor rule not augmented is too disinflationary. During the shock, the financial intermediaries are further constraint and the credit spread increases. A tighter monetary policy would mean an even tighter condition for credit. Hence the optimal reaction to the spread is slightly negative to ease constraints on financial intermediaries, as in [Curdia and Woodford 2010].

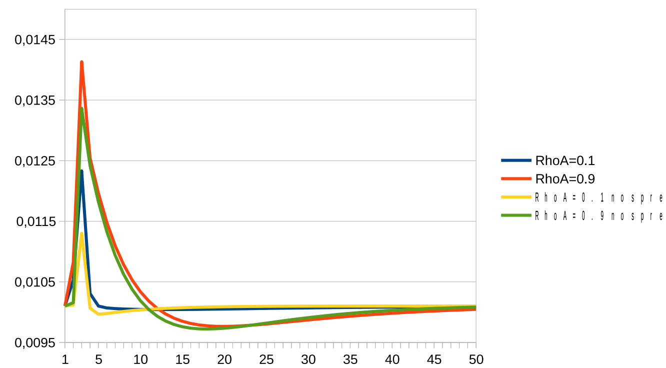

Fig. 9. Simplified Gertler and Karadi 2011, credit spread augmented (or not) Taylor rule: evolution of the key central bank rate with 2 different persistence of the technology shock ρ_A (0.1 and 0.9)

3.10.2 *Financial shocks.* As described in Section 3.3 Equation 5, the value of the start up funds ratio (ω not to be mistaken with the credit spread) a household agrees to give an entering banker will determine the steady state value of the leverage ratio (hence the constraint on the financial intermediary capital). With the steady state value of the leverage decreasing when ω faces a positive shock:

$$\phi = \frac{1 - R\theta}{\omega + \theta(R_k - R)}$$

with the equation of motion for the start up funds ratio:

$$\omega_t = \omega_{t-1}^{\rho_\omega} \omega^{1-\rho_\omega} \exp(\epsilon_t^\omega)$$

In which case, this is a financial shock and as in the work of [Curdia and Woodford 2010], when a financial shock occurs, it becomes optimal to choose a negative reaction to the credit spread increase as oppose to the findings in the case of non-financial shock (technology in our case). When setting the value of $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ and scanning the credit spread coefficient in the negative and positive range, the optimal value is $\phi_\omega = 0.017$ as shown in Figure 10.

At this stage of our investigation, we can emit a intermediary conclusion. With a cautious note that the actual values of the Taylor rule coefficients are sensitive to our calibration of the model. In line with the [Curdia and Woodford 2010], when facing non-financial shocks, the central bank should strongly positively react to inflation, not react to output and should slightly positively react to credit spread.

If the shock is financial, this intermediate conclusion is changed in so far as the central bank should slightly negatively react to the credit spread.

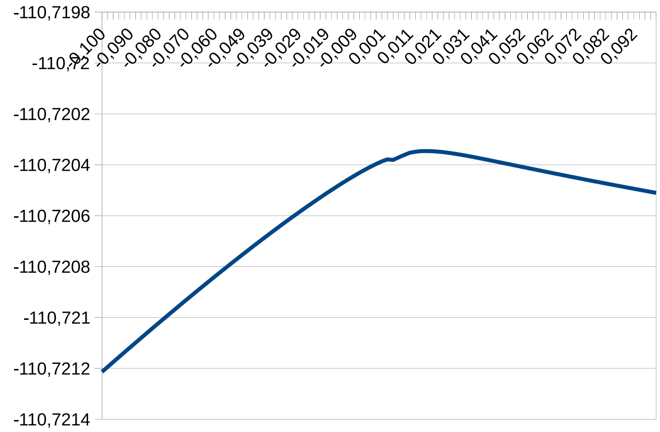

Fig. 10. Simplified Gertler and Karadi 2011, credit spread augmented Taylor rule: financial shock - welfare evolution in function of the Taylor rule credit spread coefficient ϕ_ω

4. COUNTER-CYCLICAL PRUDENTIAL LEVERAGE RATIO

In this section, we impose a leverage ratio on financial intermediaries and observe the change in the expected welfare of the households.

First we model a fixed leverage ratio, then we model an intertemporal prudential policy.

4.1 Fixed leverage ratio

The parameter λ is dropped from the originila [Gertler and Karadi 2011] and the modeling of the financial intermediaries is simplified where we assume that if a leverage ratio upper bound is imposed, this constraint will be binding. Dropping the λ parameter (which was the portion of funds a bankers could run away with) means that we are giving up the micro-foundation for an endogenous capital constraint on financial intermediaries. This is a clear negative point of this blunt prudential measure that we will address in Section 4.1.3.

From the empirical work form [Moussu and Petit-Romec 2014], if we acknowledge that

—top bankers pay depends on the profit a bank make and that only risk is rewarded to a bank,

—top bankers cannot be financially punish if their bank goes bankrupt after taking on too much risk.

This is why I take the hypothesis that the financial intermediary will maximize the risk taken ($Q_t S_{jt}$) and the prudential leverage ratio will be binding. This hypothesis is strong and might not reflect the mechanism observed. It could be more appropriate to impose a maximum limit on the leverage as in [He and Krishnamurthy 2014]. In fact, when the constraint is binding, more sever effects can occur (systemic risk crisis), that are not covered in the following simplified model. A final note, since the work of [Moussu and Petit-Romec 2014], there have been some evolution, indeed, as mentioned in The Wall Street Journal of the first of June, titled *European Banks Copy Wall Streets Move on Profit Metric*, the banks are moving from the controversial return on equity (**RoE**) metrics to **RoTE**

The measure, **return on tangible equity** strips out factors such as goodwill from acquisitions from the equation, reduces the size of the assets, and can also flatter the rate of profitability.

The modification to the [Gertler and Karadi 2011] model in the case of a simplified financial intermediaries behavior is derived in Appendix I.

We have to take those leverage ratio consideration with a grain of salt as they don't take bank's **off-balance-sheet exposure** into account (e.g. when a bank grant guarantee to a financial product that during a crisis is potentially considered as "*toxic*"), which can help explain the procyclicality, in particular in the case of precautionary hoarding as detailed in [Brunnermeier and Oehmke 2013].

When simulating with the simplified Gerlter and Karadi 2011 model, the long-term leverage ratio obtained is around 2. This is the lower bound of the empirically observed leverage ratios. In [Kalemli-Ozcana et al. 2012], they use international data on banks (200 000 observations) for 2000-2009 and measure the leverage (ratio as asset to equity). They find a mean leverage ratio of 12.4, and a minimum-maximum range of $[1.3 \quad 46.3]$. For their European bank sample, they find that the leverage increased in the run up to the crisis and plunged after the outburst as in Figure 11.

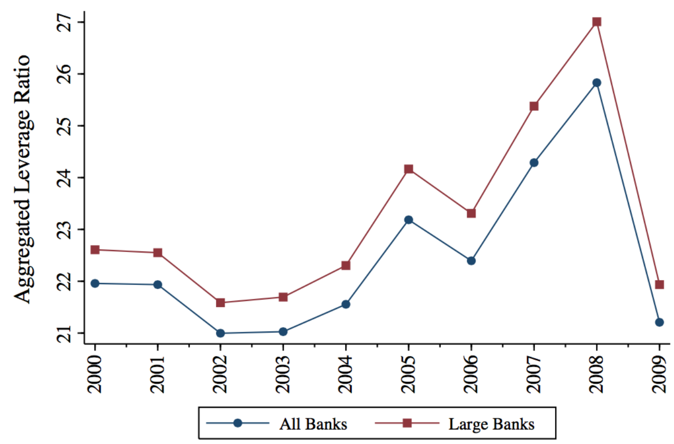

Fig. 11. [Kalemli-Ozcana et al. 2012], aggregate leverage ratio evolution for European banks

4.1.1 *"Simple" Taylor rule.* As a first step I impose a fixed leverage ratio of 0.5 and the monetary policy doesn't take the credit spread into account.

We obtain the dynamics shown in Figure 12 with a negative technology shock of variance (quarterly) 0.007. When a negative productivity shock occurs, the marginal costs for the firms increase, hence the latter have to raise their price and the inflation goes up. To fight that inflation, the monetary authority raises its key rate (the interest rate). In order to compensate for the negative shock, workers work more hours, which lower their marginal product with respect to the marginal product of capital, hence the **credit spread increases**.

4.1.2 *"Augmented" Taylor rule.* In the following, the Taylor rule is augmented to counter the credit spread with the interest rate and I "scan" for the optimal coefficient $\phi_\omega \in [0; 350]$ as in Section 3.10. I fix $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ as in the original paper. The welfare as a function of the credit spread coefficient is shown in

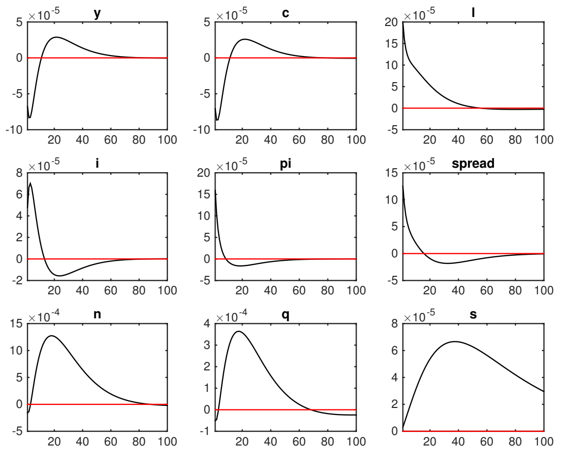

Fig. 12. Simplified Gertler and Karadi 2011, binding leverage constraint, no credit spread in the monetary policy

Figure 13. The welfare gradually degrades before increasing with a strong monetary policy reaction to the credit spread. Hence with a blunt fixed leverage ratio imposed on financial intermediary, it is beneficial for the central bank to strongly negatively react to credit spread "up to a certain point". There is no clear way of empirically determining the certain point above which the welfare would degrade, as it would depend of the calibration of each economy (which would evolve) and on the nature of the shock.

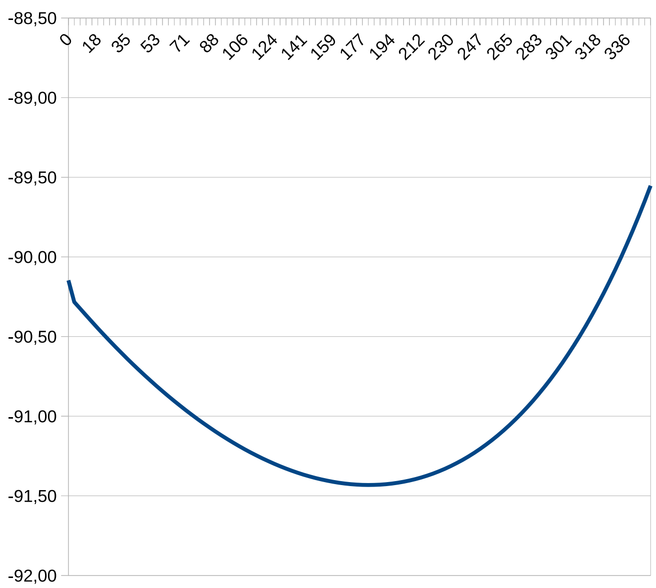

Fig. 13. Simplified Gertler and Karadi 2011, binding fixed leverage constraint, augmented Taylor rule - Welfare as a function of the credit spread coefficient

4.1.3 *"Independent" macro-prudential policy.* In the following, the Taylor rule is not "augmented" with a consideration on the credit spread. It is rather an independent prudential rule, the imposed leverage on financial intermediaries ϕ_t, that varies counter-cyclically with a long-term objective $\hat{\phi} = 0.5$. We add a smoothing factor $\rho_\phi = 0.1$ as in Equation 10.

$$\phi_t = (1-\rho_\phi)\left(\hat{\phi} + \text{range}(1 - \frac{1}{exp(\text{Spread}_t - \text{Spread})}\right) + \rho_\phi \phi_{t-1} \quad (10)$$

I have to calibrate the range for the model to converge, its value is *ad hoc*. Setting it to 0.02, in line with the recommendation in [Angelini et al. 2014a] of a maximum of 2.5% following the announcement from the regulator, I find an improvement in the welfare that would have to be matched with a augmented Taylor coefficient of the order of 300.

Hence with this simplified model, an independent macro-prudential rule is not necessary stricto census as an augmented Taylor rule that strongly reacts to the credit spread can also reduce the welfare loss in the same order of magnitude.

4.2 Direct impact on the interest rate

In this section, I seek to keep the main benefit of the paper to the literature on monetary policy: the endogenous balance sheet constraints on financial intermediaries. The main idea is for the prudential authority to impose rules to act contracyclically on the leverage ratio ϕ_t, that is the prudential authority should tighten the leverage ratio constraint in good times and relax the leverage ratio during a severe crisis.

In [Angelini et al. 2014a], they assess the impact of the Basel III regulation which is expressed in terms of equity that must be held as a percentage of risk-weighted assets. One part of this regulation is a counter-cyclical buffer imposed on banks.

This could be replicated via a control on a tax on the benefits a banker is bringing to its household $\mathcal{T}_{1t}$ or directly on the financial intermediary's return on assets $\mathcal{T}_{2t}$. This would enable the prudential authority to dynamically control the capital constraint on financial intermediaries and its steady state value:

$$\phi = \frac{1 - R\theta\mathcal{T}_2}{\omega\mathcal{T}_1 + \theta(R_k - R)\mathcal{T}_2}$$

In my model, for simplicity, the prudential authority choose to influence directly the private loans interest rates with a factor τ_t as in Equation 11 where the factor varies according to the level of the aggregate credit with respect to the output in the economy as in Equation 12. This is a simplification where we consider that there would be a tax $\mathcal{T}_{2t}$ only on stochastic return on intermediary assets that we name τ_t, and which modifies Equation 22 from Appendix F. We do not take into account distortionary effects that could lead agent to change their investment behavior. As mentioned in [Angelini et al. 2014a], varying the leverage requirement should have a cost on welfare, the Van den Heuvel formula, we do not take this either into account in this simple model.

$$\tau_t = (1-\rho_\tau)\left(\hat{\tau} - \kappa_\tau(\frac{Q_t S_t}{Y_t} - \frac{QS}{Y})\right) + \rho_\tau \tau_{t-1} \quad (11)$$

$$R_{kt+1} = \frac{P_{mt+1}\alpha\frac{Y_{t+1}}{K_{t+1}} + (Q_{t+1} - \delta)\xi_{t+1}}{Q_t}\tau_t \quad (12)$$

The prudential leverage is *leaning-against-the-wind*. Meaning that when the aggregate credit value is raising, the constraint on bank capital is getting more constraint, the stochastic return on intermediary assets is decreasing on average.

4.2.1 *Non-financial shocks.* I start with a technology shock and search for the optimal simple Taylor-rule response from the monetary authority. Noting that the model is sensitive to small variations of κ_τ, I scan for the optimal value of $\kappa_\tau \in [-0.001, 0.001]$ and $\phi_\omega \in [-0.01, 10]$. And find the optimal value for $\kappa_\tau \to 0.001$ and $\phi_\omega \to -0.01$ as shown in Figure 14. Similarly as we found in Section 3.10, when there is a technology shock, this is a non-financial shock and it is optimal for the augmented Taylor rule to positively react to the credit spread (negative ϕ_ω).

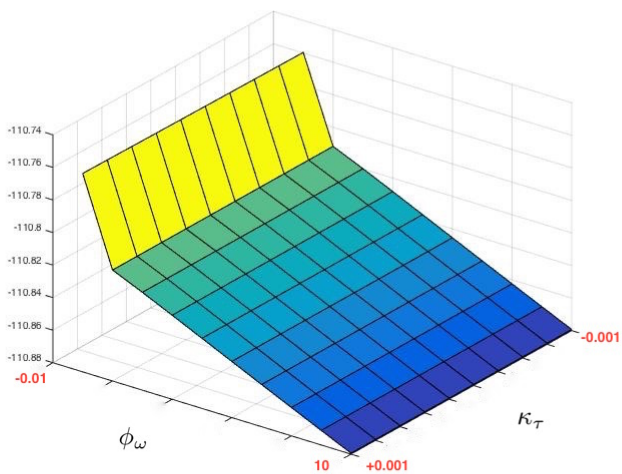

Fig. 14. Simplified Gertler and Karadi 2011, augmented Taylor rule - prudential influence on the private loan rates

In this model with endogenous capital formation for financial intermediaries, we find the opposite conclusion on the optimal credit spread coefficient in the Taylor rule as to when we impose a blunt leverage ratio on financial intermediaries. The prudential authority can positively influence the interest rate granted to private firms. Indeed, while scanning for $\phi_\pi \in [1.5; 10]$ and $\phi_\omega \in [-0.01; 10]$, we find an evolution of the agents welfare as in Figure 15 with optimal values $\phi_\pi \approx 8$ and $\phi_\omega \to -0.01$, when setting $\phi_y = 0.5/4$ and $\kappa_\tau = 0.001$.

As a final check, we scan for the inflation and output coefficients of the Taylor rule and we find similar result as with the [Gali 2008] or simplified [Gertler and Karadi 2011] models without prudential reaction functions as shown in Figure 16. And an optimal value of $\phi_\pi \approx 8$ and $\phi_y \to 0.5/4$ when setting $\phi_\omega = -0.01$ and $\kappa_\tau = 0.001$.

In both cases, we find a sharp decline of welfare in the case we use a positive ϕ_ω credit spread coefficient (so a negative reaction to an increase in the credit spread vis-à-vis its long-term value).

It is therefore best for the monetary authority to react to inflation and for an independent prudential authority to react to the aggregate credit in the economy, in the case of a non-financial shock.

4.2.2 *Financial shock.* As we did in Section 3.10, we check if our results still hold in the case of a financial shock. We use the same modeling, of a shock on the ratio ω that each period a household give to a banker to "start up" his business, which impact

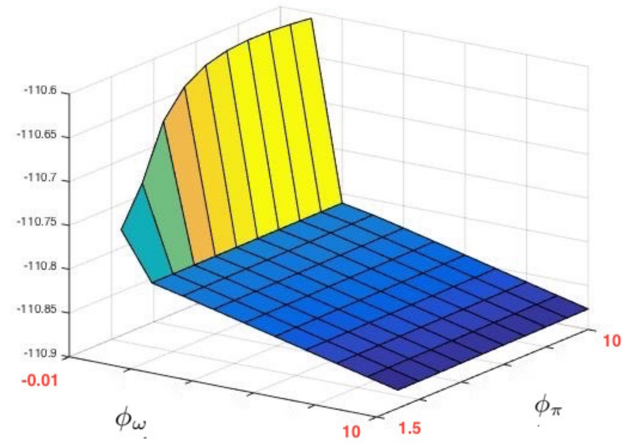

Fig. 15. Simplified Gertler and Karadi 2011, augmented Taylor rule and counter-cyclical prudential policy

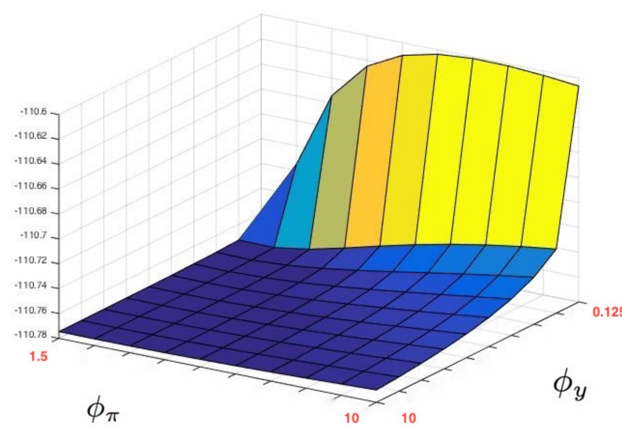

Fig. 16. Simplified Gertler and Karadi 2011, augmented Taylor rule and counter-cyclical prudential policy

the capital constraint on financial intermediaries. With such shocks, the only difference, as in Section 3.10, is that the optimal reaction of the monetary authority is to negatively react to the credit spread, with an optimal value as in Section 3.10.

4.3 Conclusion on the design of simple Taylor and Prudential rules

To conclude, if we consider that it is not possible for the authority to determine if the nature of a shock is financial (e.g. change in risk perception that will change how much start up funds households are willing to give to bankers) or if the nature of a shock is non-financial (e.g. technology shock), then it would be optimal for the monetary authority that follows a simple Taylor rule to react strongly and only to inflation and to the prudential authority to manage taxes to negatively react to the aggregate credit, so that when aggregate credit rises, the return on new credit decreases and incentives to lend to risky private firms decreases with respect to risk free (government) bonds.

5. SYSTEMIC RISK: A MICRO APPROACH

What is striking, is that as in the original paper [Gertler and Karadi 2011], there is no "default probability" for financial intermediaries that aggregate to form a systemic risk measure. So the welfare loss only goes through loans that cannot be delivered to firms and dampen the economic activity.

In this model, there is a direct effect on the maximum allowed leverage ratio to the consumer welfare. Hence when we simplify the model, we get quite a similar behavior as when the government (i.e. the central bank) directly lend to firms (but with the cost τ), it alleviate the leverage constraint ϕ_t. In the original paper [Gertler and Karadi 2011], when the central bank inflates the financial intermediaries equity N_t or the leverage ratio, it increases the incentive for the banker to divert a share of the assets $\lambda\phi_t N_t$.

In our quest to study counter-cyclical prudential action, a measure of the systemic risk was essential to assess any "positive" impact of the prudential authority to counter the build up of the systemic risk. We emulated a *leaning-against-the-wind* policy to prevent excessive accumulation of credit in the economy, where the prudential authority constrains the return of the financial intermediaries when the aggregate credit in the economy to output increases. Our method was rather blunt, to consider that systemic risk increase when the aggregate credit in the economy with respect to the output increases $\frac{Q_t S_t}{Y_t}$.

So far, our prudential policy didn't take into account (for lack of measure of) the individual *contribution* of the financial intermediary to the systemic risk. As proposed in [Brunnermeier and Oehmke 2013], we should consider an intermediary exposition to risk factors and whether the risks of a financial intermediary portfolio are *correlated* with the risks of other financial intermediaries' portfolios. This lack of systemic risk modeling is also one of the main conclusion of the literature review of [Carre et al. 2015]. This would be the object of a future work to be able to propose a robust conclusion, to add financial intermediaries risk correlations and have the prudential authority monitor the systemic risk. An approach would be to create different sectors and geographical area, and consider how, based on incentives, different bank build their portfolio and spread their risk exposure to the rest of the banking system.

6. CONCLUSION

In this work, we researched the optimal design for a simple Taylor rule that we could choose to impose on a monetary authority. Working with a simplified version of the model developed in [Gertler and Karadi 2011] and removing the micro-foundations for endogenous capital constraint on financial intermediaries, there is improvement in welfare in times of negative productivity shocks when the Taylor rule strongly reacts to variations in the credit spread. With this model, it is also beneficial to add a counter-cyclical independent prudential policy aside the monetary policy.

When keeping the micro-foundations of that model for endogenous capital constraint on financial intermediaries, the benefit on welfare when the Taylor rule reacts to the credit spread depends on the nature of the shocks, financial (e.g. change in risk perception that will change how much start up funds households are willing to give to bankers) or if the nature of a shock is non-financial (e.g. technology shock).

We modeled the intervention of the macro-prudential authority as having a direct impact on the stochastic return on financial intermediary assets, in the form of a tax. It is optimal for the macro-prudential policy to *lean-against-the-wind*, meaning that when the aggregate credit value is raising (a blunt measure of systemic risk increase), the prudential authority should set the tax for the constraint on bank capital to be more constraining, and vice versa.

As it is not possible to determine, as they occur, the nature of shocks (financial or non-financial, or even a mix), the results of our work lead us to propose that the monetary authority strongly react only to inflation and that an independent prudential authority lean against aggregate credit aggregate increase, and relieve capital constraint on financial intermediary once a crisis broke out.

Finally, our model, as the original [Gertler and Karadi 2011], doesn't consider a default probability that would vary cyclically (with the credit aggregate, or other factors), nor the correlation of risk exposures among the financial intermediaries. We haven't modeled the individual contribution of a financial intermediary to the systemic risk. As detailed in [Brunnermeier and Oehmke 2013], systemic risk builds up in the background, is difficult to notice until it bursts during a crisis (as in 1929 and 2007). It is important in future work to model the systemic risk at the micro level and its contribution to the welfare. Conclusion could vary and/or propose differently designed prudential measures.

ACKNOWLEDGMENTS

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APPENDIX

A. SIMPLE MONETARY POLICY RULES

At a minimum, the monetary policy rule has to respect the basic arbitrage between interest and inflation rate, with:

—i the interest rate on money from today to tomorrow

—r the real rate of interest, which is the relative price of good I can buy today or tomorrow

—P_1 price today

—P_2 price tomorrow

This illustrates the Fisher equation:

$$\begin{cases} 1+r = \frac{P_1(1+i)}{P_2} \\ \pi = \frac{P_2-P_1}{P_1} \\ 1+r = \frac{1+i}{1+\pi} \end{cases}$$

If we introduce agents' inflation *expectations*:

$$1+i_t = (1+E_t[\pi_{t+1}])(1+r_t) \quad (13)$$

A simple inflation based interest rate rule for the Central Bank could be as in [Gali 2008] Chapter 2:

$$i_t = \rho + \phi_\pi \pi_t \quad (14)$$

where $\phi_\pi \geq 0$ and ρ is defined as the households discount rate (the simple "mathematisation" that you prefer to consume today rather than tomorrow when you might be already dead).

Combining equations 13 and 14, then the interest rate disappears, and in case of "perfect" expectations ($E_t[\pi_{t+1}] = \pi_{t+1}$), and small variations, this simplifies into:

$$\pi_t \approx \frac{1}{\phi_\pi}\left[\pi_{t+1} + r_t - \rho\right] \quad (15)$$

The equation 15, implies that the inflation rate π_t (hence the price level) is only determinate when $\phi_\pi \leq 1$, this is the *Taylor principle*.

This rule can be extended as in [Gali 2008] Chapter 3, where now the monetary authority reacts to both inflation and the output gap (the difference between the output at time t and its steady state value):

$$i_t = \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^{\text{nat}}) \quad (16)$$

As proposed by [Taylor 2008], we can introduce a consideration on financial stability and *augment* the monetary policy rule, following [Curdia and Woodford 2009] notation:

$$\hat{i}_t = \phi_\pi \pi_t + \phi_Y \hat{Y}_t - \phi_\omega \hat{\omega}_t \quad (17)$$

—i_t interest rate on savings deposits: $\hat{i}_t = \log(\frac{1+i_t}{1+\bar{i}})$

—Y_t is a measure of the GDP: $\hat{Y}_t = \log(\frac{Y_t}{\bar{Y}})$

—ω_t is a measure of the financial stability (via the credit spread proxy, i.e. the spread between deposit rates and lending rates as in [Curdia and Woodford 2009], or via the interbanking sector proxy, the LIBOR-OIS spread as in [Taylor 2008]): $\hat{\omega}_t = \log(\frac{1+\omega_t}{1+\bar{\omega}})$

In other words, the central bank funds rate should be lowered when financial instability increases. The financial stability proxies can be measured as the spread between the Euribor and the overnight indexed swap (OIS) as illustrated in Figure 17 and 18, for the euro area. The OIS is an overnight unsecured swap rate determined by the central bank and the Euribor is an average of the interest rates at which banks lend unsecured funds in the euro area inter-bank market. The spread between the two is a proxy to measure of bank default probability. In the [Gertler and Karadi 2011] model that we use in this paper, the credit spread is expressed as in Equation 23 in Appendix F.

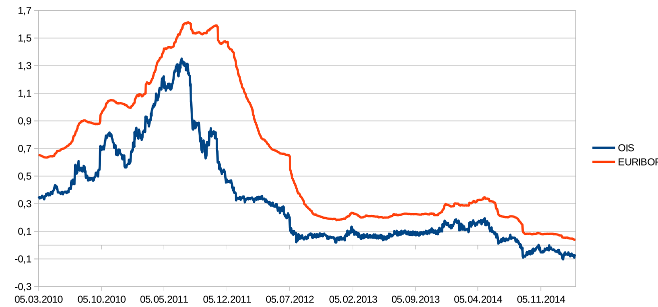

Fig. 17. Eurozone Euribor and OIS rate (source: Reuters)

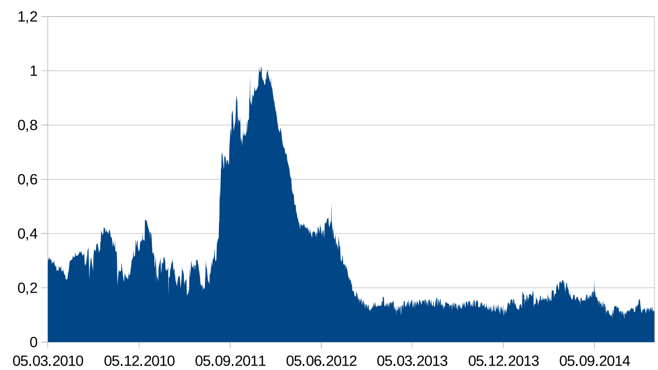

Fig. 18. Eurozone credit spread Euribor-OIS, absolute value (source: Reuters and author's calculation)

In Appendix B I present alternative monetary policy rules when the zero lower bound is binding.

B. ALTERNATIVE MONETARY POLICIES TO INFLATION TARGETING WHEN THE LOWER BOUND IS BINDING

In the previous chapter, we have reviewed monetary policies that take the inflation into account. Here are some alternatives when the zero lower bound is binding.

B.1 7/3 threshold rule

In 2011, Charles Evans proposed that the Federal Reserve should:

> hold the federal funds rate at extraordinarily low levels until the unemployment rate falls substantially, say from its current level of 9.1% to 7.5% or even 7%, as long as medium-term inflation stayed below 3%.

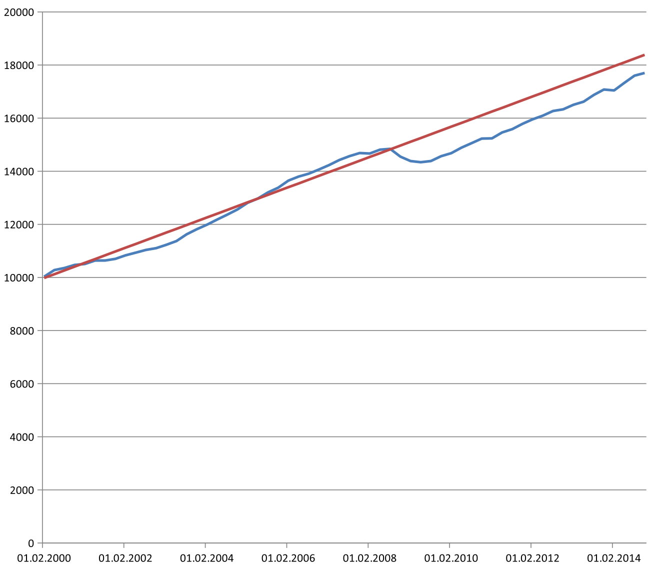

Fig. 19. US GDP versus its trend calculated over 1Q1996 to 1Q2008 (source: Datastream and trend from author's calculations)

B.2 Nominal GDP target path

Alternatively, the Federal Reserve could commit to keep its low levels of funds rate based on a nominal GDP target path as shown in Figure 19 for the US, moving back to its conventional monetary policy once a deterministic GDP target path is reached.

C. LINK TO THE MATLAB AND DYNARE CODES

Link to the Matlab and Dynare codes:
http://excellerate.fr/wp-content/uploads/2015/04/code_gk.zip

D. A MODEL À LA GALI 2008

If we log-linearize the optimality condition

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho)$$

Prodution function y_t with a_t the level of technology

$$y_t = a_t + (1-\alpha)n_t$$

The output gap

$$\tilde{y}_t \equiv y_t - y_t^{nat}$$

With

—y_t^{nat} the *natural level of output* which is the equilibrium level of output under flexible prices

$$y_t^{nat} = \Psi_{ya}^n a_t + \vartheta_y^n$$

With the constants

$$\begin{cases} \Psi_{ya}^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \\ \vartheta_y^n = -\frac{(1-\alpha)(\mu - log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} \end{cases}$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

With θ the index of price stickiness and

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}(\sigma + \frac{\varphi+\alpha}{1-\alpha})$$

and

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

Dynamic IS Curve

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^n) + E_t\tilde{y}_{t+1}$$

Monetary policy: interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t$$

The *natural rate of interest* is defined as

$$r_t^n \equiv \rho + \sigma E_t(\Delta y_{t+1}^n)$$

$$r_t^n = \rho + \sigma \Psi_{ya}^n E_t(\Delta a_{t+1})$$

Productivity shock, AR(1)

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

Then the implied natural rate of interest rate is:

$$\hat{r}_t^n \equiv r_t^n - \rho = -\sigma \Psi_{ya}^n (1-\rho_a) a_t \tag{18}$$

Following the coefficient identification method, we obtain:

$$\tilde{y}_t = -\sigma \Psi_{ya}^n (1-\rho_a)(1-\beta\rho_a)\Delta_a a_t \tag{19}$$

$$\pi_t = -\sigma \Psi_{ya}^n (1-\rho_a)\kappa \Delta_a a_t \tag{20}$$

with

$$\Delta_a = \frac{1 + \frac{\phi_y}{\sigma(1-\rho_a}}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_\pi] + \kappa(\phi_\pi - \rho_a)}$$

D.1 Second order

In order to be able to calculate the welfare, we have to derive the model at least to the ordre 2.

$$N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t}$$

$$Y_t = \frac{A_t N_t^{(1-\alpha)}}{1 + \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1-\alpha+\alpha\epsilon}{1-\alpha} var_i(p_t(i))}$$

NB: In practice, when I implement the code, I don't take into account the variance of the price $var_i(p_t(i))$.

$$A_t = A_{t-1}^{\rho_A} exp(\epsilon_t^A)$$

$$exp(-i_t) = \beta \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} \frac{P_t}{P_{t+1}}$$

I define: $exp(-i_t) \equiv 1/R_t$

$$C_t = Y_t$$

$$\pi_t = \frac{P_t}{P_{t-1}}$$

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

$$P_t^* = \frac{\sum_{k=0}^{\infty} \theta^k E_t(Q_{t,t+k} Y_{t+k,t} \mathcal{M} MC_{t+k,t})}{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} Y_{t+k,t}}$$

$$P_t^* = \frac{K_{pt}}{F_{pt}}$$

$$K_{pt} = \mathcal{M} \frac{Y_t}{P_t C_t} MC_{t,t} + \beta\theta E_t \{K_{pt+1}\}$$

$$F_{pt} = \frac{Y_t}{P_t C_t} + \beta\theta E_t \{F_{pt+1}\}$$

As the marginal cost is defined as $MC_t = \frac{\frac{\text{dCost}}{\text{dWorker}}}{\frac{\text{dOutput}}{\text{dWorker}}}$:

$$MC_{t,t} = \frac{W_t N_t^{\alpha}}{(1-\alpha)A_t}\left(1 + \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1-\alpha+\alpha\epsilon}{1-\alpha} var_i(p_t(i))\right)$$

I define the monetary policy as:

$$R_t = \frac{1}{\beta} + \phi_\pi(\pi_t - 1) + \phi_y log(Y_t/Y)$$

With

$$\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$$

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

$$Y_{t+k,t} \equiv \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

D.2 Steady state

$$R = \frac{1}{\beta}$$

$$A = 1$$

$$\pi = 1$$

$$N = \left(\frac{1-\alpha}{\mathcal{M}} A^{1-\sigma}\right)^{\frac{1}{\sigma(1-\alpha)+\varphi+\alpha}}$$

$$Y = AN^{1-\alpha}$$

$$F_p = \frac{1}{1-\beta\theta}$$

$$P = P^* = \frac{K_p}{F_p} = 1$$

$$W = PN^{\varphi} Y^{\sigma}$$

$$MC = \frac{P}{\mathcal{M}}$$

$$K_p = F_p$$

E. GERTLER AND KARADI 2010 SIMPLIFIED RESOLVED

E.1 Households

Household preferences

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right]$$

Household budget constraint

$$C_t = W_t L_t + \Pi_t + R_t B_t - B_{t+1}$$

where R_t is the bond gross real return from $t-1$ to t, B_{t+1} is the total quantity of short-term debt the household acquires (i.e. bonds), W_t the real wage, and Π_t net payouts to the household from ownership of both non-financial and financial firms. The Lagrangean writes

$$\begin{aligned}\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \big[\log(C_t) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \\ + \varrho_t (W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} - C_t)\big]\end{aligned}$$

FOC wrt consumption C_t

$$\frac{1}{C_t} = \varrho_t$$

FOC wrt labor L_t

$$\varrho_t W_t = \chi L_t^{\varphi}$$

FOC wrt bonds B_{t+1}

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = 1$$

with

$$\Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t}$$

E.2 Intermediate Goods Firms

Competitive intermediate goods producers acquire capital K_{t+1} for use in production, at price Q_t. To finance its capital acquisition the firm issues S_t claims equal to the number of units of capital acquired and prices each claim at the price Q_t. Then by arbitrage

$$Q_t K_{t+1} = Q_t S_t$$

Production is given by

$$Y_t = A_t (\xi_t K_t)^{\alpha} L_t^{1-\alpha}$$

where the shock ξ_t is meant to provide a simple source of exogenous variation in the value of capital. Let P_{mt} be the price of intermediate goods output. The Lagrangean writes

$$\begin{aligned}\mathcal{L}_t = P_{mt} Y_t - W_t L_t - (R_{kt} Q_{t-1} K_t - (Q_t - \delta) \xi_t K_t) \\ - \lambda_t [Y_t - A_t (\xi_t K_t)^{\alpha} L_t^{1-\alpha}]\end{aligned}$$

FOC wrt production Y_t

$$\lambda_t = P_{mt}$$

FOC wrt labor L_t

$$P_{mt}(1-\alpha)\frac{Y_t}{L_t} = W_t$$

FOC wrt capital K_t, as in [Christiano et al. 2014]

$$R_{kt+1} = \frac{P_{mt+1}\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t}$$

Given that the replacement price of capital that has depreciated is unity, then the value of the capital stock that is left over is given by $Q_{t+1}(1-\delta)\xi_{t+1}K_{t+1}$.

E.3 Capital Producing Firms

For the adjustment costs, following [Christiano et al. 2014], we define

$$\begin{aligned} f(x_t) &\equiv S(x_t) \\ &= \frac{1}{2}\left\{\exp\left[\sqrt{S''}(x_t - x)\right] + \exp\left[-\sqrt{S''}(x_t - x)\right] - 2\right\}\end{aligned}$$

where $x_t \equiv I_t / I_{t-1}$ and x denotes the steady state value of x_t. Note that $S(x) = S'(x) = 0$ and $S''(x) = S''$, where S'' denotes a model parameter.

E.4 Retail Firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms, that use intermediate output as the sole input. The final output composite is given by

$$Y_t = \left[\int_0^1 Y_{ft}^{(\varepsilon-1)/\varepsilon} df\right]^{\varepsilon/(\varepsilon-1)}$$

and

$$P_t = \left[\int_0^1 P_{ft}^{1-\varepsilon} df\right]^{1/(1-\varepsilon)}$$

With the following budget constraint:

$$P_t Y_t = \int_0^1 P_{ft} Y_{ft} df$$

From cost minimization by users of final output:

$$\mathcal{L}_t = [\int_0^1 Y_{ft}^{\frac{\epsilon-1}{\epsilon}} df]^{\frac{\epsilon}{\epsilon-1}} + \lambda_t [P_t Y_t - \int_0^1 P_{ft} Y_{ft} df]$$

First order condition w.r.t. Y_{ft}:

$$Y_{ft} Y_t^{-1} = \lambda_t^{-\epsilon} P_{ft}^{-\epsilon}$$

$$\int_0^1 Y_{ft} P_{ft} df = Y_t \int_0^1 \lambda_t^{-\epsilon} P_{ft}^{1-\epsilon} df$$

$$Y_t P_t = P_{ft}^{\epsilon} P_t^{1-\epsilon} Y_{ft}$$

Hence finally

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\varepsilon} Y_t$$

Retailers simply repackage intermediate output. It takes one unit of intermediate output to make a unit of retail output. The marginal cost is thus the relative intermediate output price P_{mt}. We introduce standard Calvo frictions: firms can adjust their price with probability $1-\gamma$. Retailers pricing problem, as all retail firms that can reset their prices will chose the same price ($P_{ft}^* = P_t^*$)

In fact, all firms that will be in a position to change their prices will chose the same optimal price

$$P^*_{ft} = P^*_t$$

Aggregate price

$$P_t = \left[(1-\gamma)(P^*_t)^{1-\varepsilon} + \gamma(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

If we log-linearize around the steady-state where $P_t = P^*_t$

$$\gamma\pi_t = (1-\gamma)(p^*_t - p_t)$$

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[P^*_t - P_{mt+i}\right] Y_{ft+i}$$

$$E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[P^*_t - \mu P_{mt+i}\right] Y_{ft+i} = 0$$

with

$$\mu = \frac{1}{1-1/\varepsilon}$$

We log-linearize around a zero inflation steady steady-state

$$p^*_t = \mu + (1-\gamma\beta) \sum_{i=0}^{\infty} \gamma^i \beta^i E_t(p_{mt+i})$$

$$p^*_t - p_t = \mu + (1-\gamma\beta) \sum_{i=0}^{\infty} \gamma^i \beta^i E_t(p_{mt+i} - p_{t+i}) + \sum_{i=1}^{\infty} \gamma^i \beta^i E_t(\pi_{t+i})$$

$$p^*_t - p_t = \gamma\beta E_t(\pi_{t+1}) + (1-\gamma\beta) E_t(p_{mt} - p_t) + \gamma\beta E_t(\pi^*_{t+1} - p_{t+1})$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \varpi(p_{mt} - p_t)$$

We obtain

$$\Pi_t = (\frac{P_{mt}}{P_t})^{\varpi} E_t(\Pi_{t+1})^{\beta}$$

With

$$\varpi = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}$$

E.5 Financial Intermediaries

For financial intermediaries, we make no modification from the original paper.

Most importantly for us, the moral hazard/costly enforcement constraint, where bankers can choose to divert a fraction λ of available funds from the project and instead transfer them back to the household. This leads to an endogenous capital constraint on banks where the incentive to cheat is balanced by the opportunity cost of not being able in the future to expand assets.

$$Q_t S_{jt} = \phi_t N_{jt} \tag{21}$$

E.6 Resource Constraint and Monetary Policy

E.6.1 *In normal times - as per per G& K.* Economy-wide resource constraint

$$Y_t = C_t + I_t + f\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right)(I_{nt} + I_{ss})$$

Capital accumulation

$$K_{t+1} = \xi_t K_t + I_{nt}$$

Monetary policy

$$i_t = (1-\rho)[i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y^*_t)] + \rho i_{t-1} + \varepsilon_t$$

where the link between nominal and interest rates is given by the following Fisher condition (modified wrt the time index for the return)

$$1 + i_t = R_t \frac{E_t P_{t+1}}{P_t}$$

F. EQUILIBRIUM CONDITIONS

Labor/consumption choice

$$\frac{W_t}{C_t} = \chi L^{\varphi}_t$$

Bonds Euler

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = 1$$

Stochastic discount factor

$$\Lambda_{t,t+1} = \frac{C_t}{C_{t+1}}$$

Arbitrage condition

$$K_{t+1} = S_t$$

Production function

$$Y_t = A_t(\xi_t K_t)^{\alpha} L^{1-\alpha}_t$$

Wage

$$W_t = P_{mt}(1-\alpha)\frac{Y_t}{L_t}$$

Stochastic return to intermediary assets

$$R_{kt+1} = \frac{P_{mt+1}\alpha\frac{Y_{t+1}}{K_{t+1}} + (Q_{t+1} - \delta)\xi_{t+1}}{Q_t} \tag{22}$$

For convenience, we also define the real rental rate of capital

$$r_{kt} = \alpha P_{mt} \frac{Y_t}{K_t}$$

Net investment

$$I_{nt} = I_t - \delta\xi_t K_t$$

FOC investment

$$Q_t = 1 + f(\cdot) + \frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} f'(\cdot) - \beta E_t \Lambda_{t,t+1} \left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}}\right)^2 f'(\cdot)$$

Aggregate price

$$P_t = \left[(1-\gamma)(P^*_t)^{1-\varepsilon} + \gamma(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Marginal gain to the banker of expanding assets

$$\nu_t = (1-\theta)\beta E_t \Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta\theta E_t \Lambda_{t,t+1} \frac{Q_{t+1}S_{t+1}}{Q_t S_t} \nu_{t+1}$$

Discounted value of having another unit of net worth holding assets constant

$$\eta_t = (1-\theta) + \beta\theta E_t \Lambda_{t,t+1} \frac{N_{t+1}}{N_t} \eta_{t+1}$$

Net worth

$$N_t = \theta[(R_{kt} - R_t)\phi_{t-1} + R_t]N_{t-1} + \omega Q_t S_{t-1}$$

Balance sheet identity

$$Q_t S_t = \phi_t N_t$$

Leverage

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t}$$

Resource constraint

$$Y_t = C_t + I_t + f\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right)(I_{nt} + I_{ss})$$

Capital accumulation

$$K_{t+1} = \xi_t K_t + I_{nt}$$

Monetary policy

$$i_t = (1-\rho)[i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho i_{t-1} + \varepsilon_t$$

Inflation

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

Log of inflation

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Finance premium (*credit spread*)

$$\text{spread}_t = R_{kt} - R_t \tag{23}$$

G. STEADY STATE

G.1 All Equations

Labor/consumption choice

$$\frac{W}{C} = \chi L^\varphi \tag{24}$$

Stochastic discount factor

$$\Lambda = 1 \tag{25}$$

Bonds Euler

$$R = \frac{1}{\beta} \tag{26}$$

Arbitrage condition

$$K = S \tag{27}$$

Production function

$$Y = (\xi K)^\alpha L^{1-\alpha} \tag{28}$$

Wage

$$W = P_m(1-\alpha)\frac{Y}{L} \tag{29}$$

Return to capital

$$R_k = \frac{P_m \alpha \frac{Y}{K} + (Q - \delta)\xi}{Q} \tag{30}$$

Net investment

$$I_n = I - \delta \xi K \tag{31}$$

FOC investment

$$Q = 1 \tag{32}$$

Aggregate price

$$P = P^* \tag{33}$$

Marginal gain to the banker of expanding assets

$$\nu = \frac{(1-\theta)\beta(R_k - R)}{1 - \beta\theta} \tag{34}$$

Discounted value of having another unit of net worth holding assets constant

$$\eta = \frac{1-\theta}{1-\beta\theta} \tag{35}$$

Net worth

$$N = \frac{\omega Q S}{1 - \theta[(R_k - R)\phi + R]} \tag{36}$$

Balance sheet identity

$$QS = \phi N \tag{37}$$

Leverage

$$\phi = \frac{\eta}{\lambda - \nu} \tag{38}$$

Resource constraint

$$Y = C + I \tag{39}$$

Capital accumulation

$$I_n = (1-\xi)K \tag{40}$$

Fisher condition

$$i = \frac{1}{\beta} - 1 \tag{41}$$

Inflation

$$\pi = 0 \tag{42}$$

G.2 Strategy

From equation 25

$$\Lambda = 1$$

From equation 32

$$Q = 1$$

From equation 42

$$\pi = 0$$

From equation 26

$$R = \frac{1}{\beta}$$

From equation 35

$$\eta = \frac{1-\theta}{1-\beta\theta}$$

From equations 36, 37, 34, and 38

$$R_K = \frac{(\lambda - \omega\eta)(1-\beta\theta) + \theta(\eta - \lambda)R(1-\beta\theta) + (1-\theta)\beta[R - \theta R^2]}{\theta(1-\beta\theta)\eta - \theta(1-\theta)\beta R + (1-\theta)\beta}$$

From equation 46, equation 47, and equation 48

$$P^* = \mu P_m P$$

Using equation 33

$$P = P^*$$

we obtain

$$P_m = \frac{1}{\mu}$$

With R_k and P_m, from equation 30

$$\frac{Y}{K} = \frac{\mu}{\alpha}[R_K - (1-\delta)\xi]$$

From equation 28

$$\frac{K}{L} = \left(\frac{Y}{K}\right)^{\frac{1}{\alpha-1}} (\xi)^{\frac{\alpha}{1-\alpha}}$$

Now

$$\frac{Y}{L} = \frac{Y}{K}\frac{K}{L}$$

From equation 39, using equation 31 and equation 40

$$\frac{C}{L} = \frac{Y}{L} - (1-(1-\delta)\xi)\frac{K}{L}$$

From equation 29

$$W = P_m(1-\alpha)\frac{Y}{L}$$

From equation 24

$$L = \left[\frac{W}{\chi}\left(\frac{C}{L}\right)^{-1}\right]^{\frac{1}{1+\varphi}}$$

Thus, with L

$$C = \frac{C}{L}L$$

$$K = \frac{K}{L}L$$

$$Y = \frac{Y}{L}L$$

From equation 27

$$S = K$$

From equation 31 and equation 40

$$I = (1-(1-\delta)\xi)K$$

From equation 34

$$\nu = \frac{(1-\theta)\beta(R_k - R)}{1-\beta\theta}$$

From equation 38

$$\phi = \frac{\eta}{\lambda - \nu}$$

From equation 37

$$N = \frac{S}{\phi}$$

G.3 Welfare

The welfare is written in recursive form as per [Faia and Monacelli 2007]:

$$\Omega_t = U(C_t, L_t) + \beta\Omega_{t+1}$$

H. SIMPLIFIED GERTLER AND KARADI 2010, SECOND ORDER

When deriving the model at the second order, the differences are:

H.1 Retail firms

$$E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[P_t^* - \mu P_{mt+i}\right] Y_{ft+i} = 0$$

We use the same method as in the Technical Appendix [Schmitt-Grohe and Uribe 2005] and we define two variables: K_p and F_p:

$$K_{pt} = \mu\Lambda_t Y_{ft} P_{mt} + \gamma\beta E_t K_{pt+1} \tag{43}$$

$$F_{pt} = \Lambda_t Y_{ft} + \gamma\beta E_t F_{pt+1} \tag{44}$$

Optimal price defined recursively:

$$P_t^* = \frac{K_{pt}}{F_{pt}} \tag{45}$$

H.2 Steady state

FOC price 1

$$K_p = \frac{\mu\Lambda Y_f P_m}{1-\gamma\beta} \tag{46}$$

FOC price 2

$$F_p = \frac{\Lambda Y_f}{1-\gamma\beta} \tag{47}$$

Optimal price

$$P^* = \frac{K_p}{F_p} \tag{48}$$

I. SIMPLIFIED GERTLER AND KARADI 2011 WITH PRUDENTIAL LEVERAGE ON FINANCIAL INTERMEDIARIES

The regulator (here the central bank) imposes a maximum leverage ratio on financial intermediaries. Hence,

$$\phi_t = \frac{Q_t S_t}{N_t}$$

And at the steady state, $\phi =$ a boundary.

The equity capital of the financial intermediary (banker's net worth) evolves:

$$N_{jt+1} = [(R_{kt+1} - R_{t+1})Q_t S_{jt} + R_{t+1} N_{jt}]$$

Then the strategy changes:

$$R_k = \frac{1}{\phi}[\frac{1-\omega\phi}{\theta} - R] + R$$